Almost orthogonal vectors
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Let \( N(d) \) denote the maximum size of a set of lines through the origin in \( \mathbb{R}^d \) pairwise at angle 89° or more. Since \( N(d) = d \) for \( d \leq 57 \), some are surprised to learn that \( N(d) \) is an exponential function of \( d \), as proved by Shannon long ago. This motivates us to examine more carefully spherical codes with all inner products within \( \varepsilon \) of zero.

Let \( \varepsilon = \varepsilon(d) \) be a positive decreasing function of \( d \) tending toward zero. We ask for the largest size of a set \( X \) of unit vectors in \( \mathbb{R}^d \) such that \( |\langle x, y \rangle| \leq \varepsilon(d) \) for all \( x, y \in X \) with \( x \neq y \). How large must \( \varepsilon(d) \) be in order to allow \( |X| \) to grow exponentially with \( d \)? Where does linear growth give way to quadratic growth? I have mostly questions and few answers.

In this talk, we will explore bounds and constructions for spherical codes with all inner products very close to zero. I will discuss connections to frames, association schemes and quantum information theory and I will mention new results of Bukh and Cox on the optimal value of \( \varepsilon \) in the case where \( |X| = d + k \) where \( k \) is constant.